Equilibria of Replicator Dynamic in Quantum Games.

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Abstract

An evolutionarily stable strategy (ESS) was originally defined as a static concept but later given a dynamic characterization. A well known theorem in evolutionary game theory says that an ESS is an attractor of replicator dynamic but not every attractor is an ESS. We search for a dynamic characterization of ESSs in quantum games and find that in certain asymmetric bi-matrix games evolutionary stability of attractors can change as the game switches between its two forms, one classical and other quantum.

1 Introduction

An evolutionarily stable strategy (ESS) is characterized by the condition that if all individuals choose this strategy, then no other strategy can spread in a population. In earlier papers [1, 2, 3, 4, 5] we studied evolutionary stability of Nash equilibria (NE) related to quantization of classical games. We found that in certain games evolutionary stability of NE can be changed on choice by a maneuver of entangled states used in the game. Such a possibility is interesting, especially, because a transition from a classical to a quantum form (or conversely) of a game is also achieved by a similar maneuver. For these results we used a scheme to play a quantum game proposed by Marinatto and weber [6].

Maynard Smith and Price [7] introduced the idea of an ESS essentially as a static concept. Nothing in the definition of an ESS guarantees that the dynamics of evolution in small mutational steps will necessarily converge the process of evolution to an ESS. In fact directional evolution may also become responsible for the establishment of strategies that are not evolutionarily stable [8].

What are the advantages involved in the dynamic approach towards theory of ESSs? A stated reason is that dynamic approach introduces the idea of structural stability into game theory [9]. Historically Liapunov provided a classic definition of stability of equilibria for general dynamical systems. This definition can also be adapted for the stability of a NE. A pair of strategies $(p^{\star}, q^{\star})$ is Liapunov stable when for every trajectory starting somewhere in a small neighborhood of radius $\epsilon > 0$ around a point representing the pair $(p^{\star}, q^{\star})$ another small neighborhood of radius $\delta > 0$ can be defined such that the trajectory stays in it. When every trajectory starting in a small neighborhood of radius $\sigma > 0$ around the point $(p^{\star}, q^{\star})$ converges to $(p^{\star}, q^{\star})$ the strategy pair $(p^{\star}, q^{\star})$ becomes an attractor. Trajectories are defined by the dynamic underlying the game.

Taylor and Jonker [10] introduced a dynamic into evolutionary games with the hypothesis that the growth rate of those playing each strategy is proportion to the advantage of that strategy. This hypothesis is now understood as one of many different forms of replicator dynamic [8, 11]. In simple words assume that x_i is the frequency (i.e. relative proportion) of the individuals using strategy i and $\mathbf{x}$, where $\mathbf{x}^T = [x_1, x_2 ... x_i ... x_n]$ and T represents transpose, is a vector whose components are the frequencies with $\overset{n}{\underset{i=1}{\Sigma}} x_i = 1$. Let $f_i(\mathbf{x})$ be the average payoff for using i when the population is in the state $\mathbf{x}$. Let $\overline{f} = \Sigma x_j f_j$ be the average success in the population. The replicator equation is, then, written as [12]

$$\overset{\bullet}{x_i} = x_i(f_i(\mathbf{x}) - \overline{f}) \tag{1}$$

where the dot is derivative w.r.t time. In case the payoff matrix is given as $A = (a_{ij})$ with a_{ij} being the average payoff for strategy i when the other player uses j. The average payoff for the strategy i in the population (with the assumption of random encounters of the individuals) is $(A\mathbf{x})_i = a_{i1}x_1 + ... + a_{in}x_n$ and the eq. (1) turns into

$$\overset{\bullet}{x_i} = x_i((A\mathbf{x})_i - \mathbf{x}^T A\mathbf{x}) \tag{2}$$

The population state is then given as a point in n simplex $\triangle$ [13]. The hypothesis of Taylor and Jonker [10] gives a flow on $\triangle$ whose flow lines represent the evolution of the population. In evolutionary game theory it is agreed [20] that every ESS is an attractor of the flow defined on $\triangle$ by the replicator equation (1), however, the converse does not hold: an attractor is not necessarily an ESS.

We now ask a question: is it possible that a non-ESS attractor of replicator dynamic in a classical game becomes an ESS for some quantum form of the same game. This possibility, besides strengthening our previous results about relationships between parameters of entangled states and evolutionary stability of NE, gives a dynamic ground to the relevance of the theory of ESSs in quantum games.

We main result in this paper is that the above possibility exists, indeed, in certain types of games. Quantization, thus, can change non-ESS attractor of replicator dynamic into ESS or conversely.

2 Equilibria and attractors of replicator dynamic

Early studies about the attractors of replicator dynamic by Schuster, Sigmund and Wolff [14, 15] reported the dynamics of enzymatic actions of chemicals in a mixture when their relative proportions could be changed. For example in the case of a mixture of three chemicals added in a correct order, such that corresponding initial conditions are in the basin of an interior attractor, it becomes a stable cooperative mixture of all three chemicals. But if they are added in a wrong order the initial conditions then lie in another basin and only one of the chemicals survives with others two excluded. Eigen and Schuster [14, 15, 16] also studied resulting dynamics in the evolution of macromolecules before the advent of life.

Schuster and Sigmund [17] applied the dynamic to animal behavior in Battle of Sexes game and described the evolution of strategies by treating it as a dynamical system. They wrote replicator eqs. (2) for the following general bi-matrix

$$\begin{array}{cc} & \text{Female's strategy} \\ & \begin{array}{cc} Y_1 & \qquad Y_2 \end{array} \\ \begin{array}{c} X_1 \\ X_2 \end{array} & \left[\begin{array}{cc} (a_{11}, b_{11}) & (a_{12}, b_{12}) \\ (a_{21}, b_{21}) & (a_{22}, b_{22}) \end{array} \right] \end{array} (3)$$

$$\begin{array}{ccc} & & \text{Female's strategy} \\ & & \begin{array}{cc} Y_1 & \qquad Y_2 \end{array} \\ \text{Male's strategy} & \begin{array}{c} X_1 \\ X_2 \end{array} & \left[\begin{array}{cc} (a_{11}, b_{11}) & (a_{12}, b_{12}) \\ (a_{21}, b_{21}) & (a_{22}, b_{22}) \end{array} \right] \end{array}$$

where a male can play pure strategies X_1, X_2 and a female can play pure strategies Y_1, Y_2 respectively. Let in a population engaged in this game the frequencies of X_1 and X_2 are x_1 and x_2 respectively. Similarly the frequencies of Y_1 and Y_2 are y_1 and y_2 respectively. Obviously

$$\begin{aligned} x_1 + x_2 &= y_1 + y_2 = 1 \\ \text{where } x_i &\geq 0, y_i \geq 0, \text{ for } i = 1, 2 \end{aligned} \tag{4}$$

the replicator equations (2) for the matrix (2) with conditions (4) are, then, written as

$$
\begin{aligned}
\overset{\bullet}{x} &= x(1-x)\left\{y(a_{11}-a_{12}-a_{21}+a_{22})+(a_{12}-a_{22})\right\} \\
\overset{\bullet}{y} &= y(1-y)\left\{x(b_{11}-b_{12}-b_{21}+b_{22})+(b_{12}-b_{22})\right\}
\end{aligned} \tag{5}
$$

where $x_1 = x$ and $y_1 = y$. These equations are of Lotka-Volterra type describing the evolution of two populations of predator and prey [18]. Schuster and Sigmund [17] simplified the problem by taking

$$
\begin{aligned}
a_{11} &= b_{11} = a_{22} = b_{22} = 0 \\
a_{12} &= a \quad a_{21} = b \quad \text{and} \\
b_{12} &= c \quad b_{21} = d
\end{aligned} \tag{6}
$$

this does not restrict the generality and the replicator eqs. (5) remain similar. Payoffs to the male $P_M(p,q)$ and to the female $P_F(p,q)$ when the male plays X_1 with probability p (i.e. he plays X_2 with the probability $(1-p)$) and the female plays Y_1 with the probability q (i.e. she plays Y_2 with the probability $(1-q)$) are written as [19]

$$
\begin{aligned}
P_M(p,q) &= \mathbf{p}^T\mathbf{M}\mathbf{q} \\
P_F(p,q) &= \mathbf{p}^T\mathbf{F}\mathbf{q}
\end{aligned} \tag{7}
$$

where $\mathbf{M} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} p \\ 1-p \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} q \\ 1-q \end{bmatrix}$ and T is for transpose.

Now a quantum form of the matrix game (2) is played using Marinatto and Weber's scheme [6]. The players have at their disposal following initial entangled state

$$
|\psi_{ini}\rangle = c_{11}\,|11\rangle + c_{12}\,|12\rangle + c_{21}\,|21\rangle + c_{22}\,|22\rangle \tag{8}
$$

with normalization

$$
|c_{11}|^2 + |c_{12}|^2 + |c_{21}|^2 + |c_{22}|^2 = 1 \tag{9}
$$

where 1 corresponds to the pure strategies X_1 or Y_1 whereas 2 correspond to the pure strategies X_2 or Y_2. The constants c_{ij} for $i,j = 1,2$ are complex numbers in general. Players apply unitary operators on the entangled state with classical probabilities and payoffs to them are decided later by a measurement on final state [6]. The male now applies the identity operator I with probability p on

$|\psi_{ini}\rangle$ and female apply I with probability q on $|\psi_{ini}\rangle$. Payoffs to both players are written [5] in a similar form as in the eq. (7)

$$\begin{aligned} P_M(p,q) &= \mathbf{p}^T \omega \mathbf{q} \\ P_F(p,q) &= \mathbf{p}^T \chi \mathbf{q} \end{aligned} \tag{10}$$

ω and χ are quantum forms [5] of the payoff matrices $\mathbf{M}$ and $\mathbf{F}$ respectively i.e.

$$\text{and} \quad \chi = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix} (11)$$

$$\omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \quad \text{and} \quad \chi = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix}$$

where

$$\begin{aligned} \omega_{11} &= a_{11} |c_{11}|^2 + a_{12} |c_{12}|^2 + a_{21} |c_{21}|^2 + a_{22} |c_{22}|^2 \\ \omega_{12} &= a_{11} |c_{12}|^2 + a_{12} |c_{11}|^2 + a_{21} |c_{22}|^2 + a_{22} |c_{21}|^2 \\ \omega_{21} &= a_{11} |c_{21}|^2 + a_{12} |c_{22}|^2 + a_{21} |c_{11}|^2 + a_{22} |c_{12}|^2 \\ \omega_{22} &= a_{11} |c_{22}|^2 + a_{12} |c_{21}|^2 + a_{21} |c_{12}|^2 + a_{22} |c_{11}|^2 \end{aligned} \tag{12}$$

similarly

$$\begin{aligned} \chi_{11} &= b_{11} |c_{11}|^2 + b_{12} |c_{12}|^2 + b_{21} |c_{21}|^2 + b_{22} |c_{22}|^2 \\ \chi_{12} &= b_{11} |c_{12}|^2 + b_{12} |c_{11}|^2 + b_{21} |c_{22}|^2 + b_{22} |c_{21}|^2 \\ \chi_{21} &= b_{11} |c_{21}|^2 + b_{12} |c_{22}|^2 + b_{21} |c_{11}|^2 + b_{22} |c_{12}|^2 \\ \chi_{22} &= b_{11} |c_{22}|^2 + b_{12} |c_{21}|^2 + b_{21} |c_{12}|^2 + b_{22} |c_{11}|^2 \end{aligned} \tag{13}$$

For initial unentangled state $|\psi_{ini}\rangle = |11\rangle$ the matrices ω and χ are same as $\mathbf{M}$ and $\mathbf{F}$ respectively. The classical game is, therefore, embedded in the quantum game. Simplified matrices ω and χ can be obtained by the assumption of eq. (6) i.e.

$$\begin{aligned} \omega_{11} &= a |c_{12}|^2 + b |c_{21}|^2, \quad \omega_{12} = a |c_{11}|^2 + b |c_{22}|^2 \\ \omega_{21} &= a |c_{22}|^2 + b |c_{11}|^2, \quad \omega_{22} = a |c_{21}|^2 + b |c_{12}|^2 \\ \chi_{11} &= c |c_{12}|^2 + d |c_{21}|^2, \quad \chi_{12} = c |c_{11}|^2 + d |c_{22}|^2 \\ \chi_{21} &= c |c_{22}|^2 + d |c_{11}|^2, \quad \chi_{22} = c |c_{21}|^2 + d |c_{12}|^2 \end{aligned} \tag{14}$$

the replicator eqs. (5) can now be written in the following 'quantum' form

$$
\begin{aligned}
\overset{\bullet}{x} &= x(1-x)[aK_1 + bK_2 - (a+b)(K_1+K_2)y] \\
\overset{\bullet}{y} &= y(1-y)[cK_1 + dK_2 - (c+d)(K_1+K_2)x]
\end{aligned} \tag{15}
$$

where $K_1 = |c_{11}|^2 - |c_{21}|^2$ and $K_2 = |c_{22}|^2 - |c_{12}|^2$. These equations reduce to eqs. (5) for $|\psi_{ini}\rangle = |11\rangle$ i.e. $|c_{11}|^2 = 1$. Similar to classical version [17] the dynamic (15) has five rest or equilibrium points $x = 0, y = 0$; $x = 0, y = 1$; $x = 1, y = 0$; $x = 1, y = 1$; and an interior equilibrium point

$$
x = cK_1 + dK_2(c+d)(K_1+K_2), \quad y = cK_1 + dK_2(c+d)(K_1+K_2) \tag{16}
$$

This equilibrium point becomes same as in the classical game [17] for $|\psi_{ini}\rangle = |11\rangle$ i.e.

$$
x = cc + d, \quad y = aa + b \tag{17}
$$

We use the method of linear approximation [18] at equilibrium points to find the general character of phase diagram of the system (15). Write the system (15) as

$$
\overset{\bullet}{x} = \mathbf{X}(x,y), \quad \overset{\bullet}{y} = \mathbf{Y}(x,y) \tag{18}
$$

the matrix for linearization [18], to be evaluated at each equilibrium point in turn, is

(19)

$$
\begin{bmatrix} \mathbf{X}_x & \mathbf{X}_y \\ \mathbf{Y}_x & \mathbf{Y}_x \end{bmatrix}
$$

where, for example, $\mathbf{X}_x$ denotes $\partial\mathbf{X}\partial x$. Write now these terms as

$$
\begin{aligned}
\mathbf{X}_x &= (1-2x)\{aK_1 + bK_2 - (a+b)(K_1+K_2)y\} \\
\mathbf{X}_y &= -x(1-x)(a+b)(K_1+K_2) \\
\mathbf{Y}_x &= -y(1-y)(c+d)(K_1+K_2) \\
\mathbf{Y}_y &= (1-2y)\{cK_1 + dK_2 - (c+d)(K_1+K_2)x\}
\end{aligned} \tag{20}
$$

the characteristic equation [18] at an equilibrium point is obtained from

$$= 0(21)$$

$$\left| \begin{array}{cc} (\mathbf{X}_x - \lambda) & \mathbf{X}_y \\ \mathbf{Y}_x & (\mathbf{Y}_y - \lambda) \end{array} \right| = 0$$

The patterns of phase paths around equilibrium points classify the points into a few principal cases. Suppose λ_1, λ_2 are roots of the characteristic eq. (2). A few cases are as follows:

(1): λ_1, λ_2 real, different, non-zero, and same sign. If $\lambda_1, \lambda_2 > 0$ then the equilibrium point is an unstable node or a repellor. If $\lambda_1, \lambda_2 < 0$ the node is stable or an attractor.

(2): λ_1, λ_2 real, different, non-zero, and opposite sign. The equilibrium point is a saddle.

(3): $\lambda_1 = \lambda_2 = \alpha + i\beta$, $\beta \neq 0$ The equilibrium is a stable spiral (attractor) if $\alpha < 0$, an unstable spiral (repellor) if $\alpha > 0$, a centre if $\alpha = 0$.

Consider an equilibrium or rest point $x = 1, y = 0$ written simply as $(1, 0)$. At this point the characteristic eq. (2) has these roots

$$\lambda_1 = -aK_1 - bK_2, \quad \lambda_2 = -cK_2 - dK_1 \tag{22}$$

For the classical game, i.e. $|\psi_{ini}\rangle = |11\rangle$, these roots are $\lambda_1 = -a$, $\lambda_2 = -d$. Therefore in case $a, d > 0$ the equilibrium point $(1, 0)$ is an attractor in the classical game. Every ESS is also an attractor but the converse is not true. We now ask if the attractor $(1, 0)$ is an ESS as well or not. The game of the matrix (2) with simplifications given in eq. (6) is an asymmetric game and the equilibrium $(1, 0)$ is an ESS if it is a strict NE [20]. The strict NE conditions for the point $(1, 0)$ are

$$\begin{aligned} P_M(1,0) - P_M(p,0) &= (1-p)\{a(|c_{11}|^2 - |c_{21}|^2) + \\ b(|c_{22}|^2 - |c_{12}|^2)\} &> 0 \\ P_F(1,0) - P_F(1,q) &= q\{c(|c_{11}|^2 - |c_{12}|^2) + \\ d(|c_{22}|^2 - |c_{21}|^2)\} &> 0 \end{aligned} \tag{23}$$

for all $p, q \in [0, 1]$ with $p \neq 1$ and $q \neq 0$. In classical game, therefore, $(1, 0)$ is an ESS when both $a, c > 0$. A comparison of the strict inequalities (23) with the roots (22) of the characteristic eq. (2) show that in case $|c_{11}|^2 = |c_{22}|^2$ the inequalities (23) guarantee that both λ_1 and λ_2 are negative and consequently an ESS is an attractor and an attractor is an ESS.

We study three cases:

(a): The equilibrium point $(1,0)$ is an attractor in classical as well as a quantum form of the game. However it is non-ESS in classical game and is an ESS in the quantum game.

(b): Point $(1,0)$ is an attractor in classical as well as a quantum game. However, it is an ESS in classical game and non-ESS in the quantum game.

(c): An interior point is a saddle (center) in the classical game but it becomes a centre (saddle) in a quantum form of the game.

2.1 Case (a)

Let the constants a, b, c and d are such that $a, d > 0$ and $b, c < 0$. The equilibrium point $(1,0)$ is, then, a non-ESS attractor in classical game. Select the parameters of the initial state c_{ij} such that $|c_{21}|^2 < |c_{22}|^2 < |c_{11}|^2 < |c_{12}|^2$ with the normalization in eq. (9). The equilibrium $(1,0)$ is now an ESS in the quantum form of the game.

2.2 Case (b)

In case $a, c, d > 0$ and $b < 0$ the point $(1,0)$ is an ESS attractor of the classical game. Select now the parameters c_{ij} of the entangled initial state such that $|c_{22}|^2 < |c_{21}|^2 < |c_{11}|^2 < |c_{12}|^2$ and $c(|c_{12}|^2 - |c_{22}|^2) < d(|c_{11}|^2 - |c_{21}|^2)$. The equilibrium $(1,0)$ is a non-ESS attractor of the corresponding quantum game.

2.3 Case (c)

At the interior equilibrium point (x, y) of eq. (16) the terms of the matrix of linearization of eq. (20) are

$$\begin{aligned}
\mathbf{X}_x &= 0, \quad \mathbf{Y}_y = 0 \\
\mathbf{X}_y &= -(cK_1 + dK_2)(cK_2 + dK_1)(a+b)(c+d)^2(K_1 + K_2) \\
\mathbf{Y}_x &= -(aK_1 + bK_2)(aK_2 + bK_1)(c+d)(a+b)^2(K_1 + K_2)
\end{aligned} \tag{24}$$

the roots of the characteristic eq. (2) are numbers $\pm\lambda$ where

$$\lambda = \sqrt{(aK_1 + bK_2)(aK_2 + bK_1)(cK_1 + dK_2)(cK_2 + dK_1)(a+b)(c+d)(K_1 + K_2)^2} \tag{25}$$

the term in square root can be a positive or negative real number. Therefore, a saddle (center) in classical game can be a center (saddle) in certain quantum form of the game. A saddle or a center in a classical (quantum) game can not be, however, an attractor or a repellor in quantum (classical) form of the game.

3 Discussion

In classical evolutionary game theory attractors of a dynamic and ESSs are usually studied with reference to population models. Extending these ideas to quantum settings requires an assumption of a population of individuals having access to quantum mechanical operators and entangled states. What is the possible relevance of such an assumption in real world? Evolutionary quantum computation (EQC) [21] is such an example. In EQC an ensemble of quantum subsystems is considered changing continually such a way as to optimize some measure of emergent patterns between the system and its environment. This optimization can thought to be related to equilibria and even to some stability property of the equilibria. Nature of quantum interaction deciding stability of equilibria imply that optimization itself depends on it. Brain itself has been proposed as an evolutionary quantum computer.

Has the ESS idea a relevance only in population models? For two players case a meaning of ESS exists when the usual term 'frequency' is replaced with 'fraction of the total time'. Two quantum interacting molecules modelled as players in a game will involve considerations of evolutionary stability and how it depends on the interaction pattern.

However our major motivation for a study of the ESS theory in quantum games is that ESS theory for classical matrix games is developed quite rigorously that also allows its almost straightforward extension into quantum games. For this purpose we found very appropriate the scheme proposed by Marinatto and Weber [6] to play a quantum game. In classical ESS theory pure strategies can be combined with probabilities that sum up to one. Similar things happen in the scheme to play a quantum game. Nevertheless, quantum aspect gives a classical matrix game more 'dimensions' and in our preferred scheme stability properties of NE and also attractors can be studied by having a control on the parameters of entangled state used in the game. ESS idea is extended to quantum games as a static concept [1] but we showed in this paper that it can be dynamically characterized as well in those games. It then provides an alternative way as well to the study of dynamic quantum games.

An important aspect by which evolutionary game theory is different from classical game theory is the role and need of rational decision makers [11]. Classical game theory was developed under the assumptions of rational decision makers. On the other hand in evolutionary game theory an individual's 'strategy' is an inherited trait usually called a 'phenotype'.A population is an abstract entity of interacting individuals with their strategies being genetically determined. This approach makes unnecessary the need for rational decision makers. In our effort to extend the ideas of evolutionary game theory, especially the idea of ESS that is central to the theory, to quantum games we do not assume a role for rationality and of decision makers in-charge of applying quantum mechanical operators on entangled states. Such decisions can be made, for example, in a group of interacting molecules without an assumption of consciousness associated with them.

We believe quantum game theory can provide a role for quantum mechanics in self organization of interacting molecules. Quantum mechanics is long known to play role in keeping the atoms together in molecules. We believe that quantum game theory paves the way for an equally important role for quantum mechanics in evolution and development of self organization and complexity in molecular systems. This aspect arises exciting new questions about quantum role in origin of life and also in origin of consciousness.

The ESS idea in population biology was developed in an attempt to understand complex behaviors in animal societies. The goal was to model evolutionary processes in populations of interacting individuals and to explain why certain states in the population are stable against perturbations induced by mutations. We do not see why the ESSs and other concepts of dynamic stability of equilibria should be useless in the context of the rise of self organization in groups of interacting molecules. Our results show that quantum mechanics has strong and important roles in selection of stable solutions in a system of interacting 'entities'. These entities can be individuals having access to quantum unitary operators and entangled states or simply a collection of molecules. We believe that if stability of solutions or equilibria can be affected by quantum interactions then it provides a new approach towards theories of rise of complexity in groups of quantum interacting entities.

Out of two perspectives on what should be an outcome of evolution matrix game theory provides one and the other is provided by optimization models [22]. In evolutionary matrix games a frequency-dependent selection takes place and all alternative strategies become equally fit when ESS establishes itself. On the other hand in optimization models the selection is frequency-independent and evolution is imagined as a hill-climbing process. Optimal solution is obtained where fitness is maximized. Evolutionary optimization is the basis of evolutionary and genetic algorithms and forms a different approach than ESSs in matrix games. These are not, however, in direct contradiction and give different outlooks on evolutionary process. We believe evolutionary optimization is another area where a role for quantum mechanics exists and quantum game theory provides hints to find it.

4 Conclusion

In this paper using Marinatto and Weber's scheme [6] to play a quantum game we explored how a maneuver of parameters of entangled state used in the game can give or take away evolutionary stability to attractors of replicator dynamic that we assumed as the underlying process of the game. We also considered the effects of such a maneuver on a saddle or a center of the dynamic. Because a classical game corresponds to a situation when an unentangled initial state is used in the game the indicated maneuver of the entangled state allows evolutionary stability coming to or going away from an attractor of the dynamic as the game changes its form from classical to quantum or conversely. These results give a dynamic characterization to our previous results that treated the ESS idea as a static concept. We suggest these results can be of interest in evolutionary quantum computing and in evolutionary optimization that also involve quantum interactions between ‘entities’ constituting a population.

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